

# Pairs-Production of Higgs in Association with Bottom Quarks Pairs at $e^+e^-$ Colliders

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## Abstract

In a previous paper, we studied the Higgs pair production in the standard model with the reaction  $e^+e^- \rightarrow t\bar{t}HH$ . Based on this, we study the Higgs pair production via  $e^+e^- \rightarrow b\bar{b}HH$ . We evaluate the total cross section of  $b\bar{b}HH$  and calculate the number total of events considering the complete set of Feynman diagrams at tree-level, and compare this process with the process  $e^+e^- \rightarrow t\bar{t}HH$ . The numerical computation is done for the energy which is expected to be available at a possible Next Linear  $e^+e^-$  Collider with a center-of-mass energy 800, 1000, 1600  $GeV$  and luminosity  $1000 fb^{-1}$ .

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## I. INTRODUCTION

Up to now the Standard Model (SM) [1] has passed all accelerator-based experimental tests. It is able to reproduce all experimental data obtained at high energy  $e^+e^-$ ,  $pp$  and  $e^\pm p$  colliders. In particular, the precision data of Large Electron Positron Collider (LEP) has verified the SM predictions with very high accuracy, and the experimental errors are in the range of about 0.1% – 1%. On the theoretical side, most of the one-loop corrections to the prominent observables have been calculated. In some cases, the leading two-loop corrections are also known. The theoretical errors are in the 0.1% – 1% range as well.

While the gauge sector of the SM has been extremely well-tested, our theoretical ideas about electroweak symmetry breaking are still not completely convincing. In fact, clarifying the mechanism of electroweak symmetry breaking will be the central problem we will have to solve with the next generation of high energy colliders. In the SM, electroweak symmetry breaking is achieved by the Higgs mechanism. The scalar Higgs boson, also predicted by this mechanism, has not been found so far.

The search for Higgs bosons is one of the principal missions of present and future high-energy colliders. The observation of this particle is of major importance for the present understanding of the interactions of the fundamental particles. Indeed, in order to accommodate the well established electromagnetic and weak interaction phenomena, the existence of at least one isodoublet scalar field to generate fermion and weak gauge bosons masses is required. The SM makes use of one isodoublet field consisting of three Goldstone bosons among the four degrees of freedom which are absorbed to build up the longitudinal components of the massive  $W^\pm$ ,  $Z^0$  gauge bosons; one degree of freedom is left over corresponding to a physical scalar particle, which is the Higgs boson [2]. Despite its numerous successes in explaining the present data, the SM cannot be completely tested before this particle has been experimentally observed and its fundamental properties studied. In particular, the Higgs boson self-interaction.

In the SM, the profile of the Higgs particle is uniquely determined once its mass  $M_H$  is fixed [3]; the decay width and branching, as well as the production cross sections, are given by the strength of the Yukawa couplings to fermions and gauge bosons, which is set by the masses of these particles. However, the Higgs boson mass is a free parameter and there are two experimental constraints on this free parameter.

The SM Higgs boson has been searched by LEP in the Higgs-strahlung process  $e^+e^- \rightarrow HZ$  for c.m. energies up to  $\sqrt{s} = 209 \text{ GeV}$  and with a large collected luminosity. In the summer of 2002, the final results of the four LEP collaborations were published and some changes were made with respect to the original publication. In particular, the inclusion of more statistics, the revision of backgrounds, and the reassessment of systematic errors. When these results are combined, an upper limit  $M_H \geq 114.4 \text{ GeV}$  is established at the 95% confidence level [4]. However, this upper limit, in the absence of additional events with respect to SM predictions, was expected to be  $M_H > 115.3 \text{ GeV}$ ; the reason is that there is a  $1.7\sigma$  excess [ compared to the value  $2.9\sigma$  reported at the end of 2000 ] of events for a Higgs boson mass in the vicinity of  $M_H = 116 \text{ GeV}$  [4].

The second constraint comes from the accuracy of the electroweak observables measured at LEP, SLAC Large Detector (SLC), and the Fermilab Tevatron, which provide sensitivity to  $M_H$ . The Higgs boson contribute logarithmically,  $\propto \log(\frac{M_H}{M_W})$ , to the radiative corrections

to the  $W/Z$  boson propagators. The status, as found in Summer 2002, is summarized in Reference [5]. When all available data (i.e. the  $Z^0$ -boson pole LEP and SLC data, the measurement of the  $W$  boson mass and total width, the top-quark mass and the controversial NuTeV result) is taken into account, one obtains a Higgs boson mass of  $M_H = 81^{+42}_{-33} \text{ GeV}$ , leading to a 95% confidence level upper limit of  $M_H < 193 \text{ GeV}$  [5].

The trilinear Higgs self-coupling can be measured directly in pair-production of Higgs particles at hadron and high-energy  $e^+e^-$  linear colliders. Higgs pairs can be produced through double Higgs-strahlung of  $W$  or  $Z$  bosons [6–9],  $WW$  or  $ZZ$  fusion [7,10–13]; moreover, through gluon-gluon fusion in  $pp$  collisions [14–16] and high-energy  $\gamma\gamma$  fusion [7,10,17] at photon colliders. The two main processes at  $e^+e^-$  colliders are double Higgs-strahlung and  $WW$  fusion:

$$\begin{aligned} \text{double Higgs-strahlung} : e^+e^- &\rightarrow ZHH \\ WW \text{ double-Higgs fusion} : e^+e^- &\rightarrow \bar{\nu}_e\nu_e HH. \end{aligned} \tag{1}$$

The  $ZZ$  fusion process of Higgs pairs is suppressed by an order of magnitude because the electron- $Z$  coupling is small. However, the process  $e^+e^- \rightarrow ZHH$  has been studied [6–9] extensively. This three-body process is important because it is sensitive to Yukawa couplings. The inclusion of four-body processes with heavy fermions  $f$ ,  $e^+e^- \rightarrow f\bar{f}HH$ , in which the SM Higgs boson is radiated by a  $b(\bar{b})$  quark at future  $e^+e^-$  colliders [18–21] with a c.m. energy in the range of 800 to 1600  $\text{GeV}$ , as in the case of DESY TeV Energy Superconducting Linear Accelerator (TESLA) machine [22], is necessary in order to know its impact on the three-body mode processes and also to search for new relations that could have a clear signature of the Higgs boson production.

The Higgs coupling with top quarks, which is the largest coupling in the SM, is directly accessible in the process where the Higgs boson is radiated off top quarks,  $e^+e^- \rightarrow t\bar{t}HH$ , followed by the process  $e^+e^- \rightarrow b\bar{b}HH$ . These processes depend on the Higgs boson triple self-coupling, which could lead us to obtain the first non-trivial information on the Higgs potential. We are interested in finding regions that allow the observation of the process  $b\bar{b}HH$  at the next generation of high energy  $e^+e^-$  linear colliders. We consider the complete set of Feynman diagrams at tree-level (Fig. 1) and used the CALCHEP [23] packages for the evaluation of the amplitudes and of the cross section.

This paper is organized as follows: In Sec. II, we present the total cross section for the process  $e^+e^- \rightarrow b\bar{b}HH$  at next generation linear  $e^+e^-$  colliders, and compare it with  $e^+e^- \rightarrow t\bar{t}HH$ . In Sec. III, we give our conclusions.

## II. DOUBLE HIGGS PRODUCTION CROSS SECTION IN THE SM AT NEXT GENERATION LINEAR POSITRON-ELECTRON COLLIDERS

In this section we present numerical result for  $e^+e^- \rightarrow b\bar{b}HH$  with double Higgs production and compare it with the process  $e^+e^- \rightarrow t\bar{t}HH$ . We carry out the calculations using the framework of the Standard Model at next generation linear  $e^+e^-$  colliders. We used CALCHEP [23] packages for calculations of the matrix elements and cross sections. These packages provide automatic computation of the cross sections and distributions in the SM as well as their extensions at the tree level. Both processes  $e^+e^- \rightarrow b\bar{b}HH$  and  $e^+e^- \rightarrow t\bar{t}HH$

are estimated, including a complete set of Feynman diagrams for  $e^+e^- \rightarrow b\bar{b}HH$ . We consider the high energy stage of a possible Next Linear  $e^+e^-$  Collider with  $\sqrt{s} = 800, 1000, 1600$   $GeV$  and design luminosity  $1000 fb^{-1}$ .

For the SM parameters, we have adopted the following: the Weinber angle  $\sin^2 \theta_W = 0.232$ , the mass ( $m_b = 4.5$   $GeV$ ) of the bottom quark, the mass ( $m_t = 175$   $GeV$ ) of the top quark, and the mass ( $m_{Z^0} = 91.2$   $GeV$ ) of the  $Z^0$ , having taken the mass  $M_H$  of the Higgs boson as input [24].

In order to illustrate our results of the production of Higgs pairs in the SM, we present a plot for the total cross section as a function of Higgs boson mass  $M_H$  for both processes  $e^+e^- \rightarrow b\bar{b}HH(t\bar{t}HH)$  in Fig. 2. We observe in this figure that the total cross section for the double Higgs production of  $b\bar{b}HH$  and  $t\bar{t}HH$  is of the order of  $0.03 fb$  for Higgs masses in the lower part of the intermediate range. The cross sections are at the level of a fraction of femtobarn, and they quickly drop as they approach the kinematic limit. Under these conditions, it would be very difficult to extract any useful information about the Higgs self-coupling from the studied process unless the  $e^+e^-$  machine works with very high luminosity.

The cross section for double Higgs boson production in the intermediate mass range is presented in Figs. 3 and 4 for total  $e^+e^-$  energies of  $\sqrt{s} = 1000, 1600$   $GeV$ . The cross sections are shown for unpolarized electrons and positrons beams. As in the case shown in Fig. 2, the cross section is at the level of a fraction of femtobarn and decreases with rising energy beyond the threshold region. However, the cross section increases with rising self-coupling in the vicinity of the SM value. The sensitivity to the  $HHH$  self-coupling is demonstrated in Ref. [25] for  $\sqrt{s} = 800$   $GeV$  and  $M_H = 130$   $GeV$  by varying the trilinear coupling  $\kappa\lambda_{HHH}$  within the range  $\kappa = -1$  and  $+2$ .

We observe in Figs. 2-4 that the cross section decreases as energy increases, but still far enough from the threshold. At some given energy, the total cross sections has its maximum value of  $\sigma_{max}^{Tot}$  depending on the Higgs mass. Since fermion chirality is conserved at the  $Z^0 - fermion$  vertex, the cross section may increase by almost double when electrons and positrons are polarized.

Fig. 5 shows the total cross section as a function of the center-of-mass energy  $\sqrt{s}$  for one representative value of the Higgs mass  $M_H = 130$   $GeV$ . We observe that the cross section is very sensitive to the Higgs boson mass and decreases when  $M_H$  increases. Our conclusion is that for an intermediate Higgs boson, a visible number of events would be produced, as illustrated in Table I.

For center-of-mass energies of  $800-1600$   $GeV$  and high luminosity, the possibility of observing the processes  $b\bar{b}HH$  and  $t\bar{t}HH$  are promising as shown in Table I. Thus, a high-luminosity  $e^+e^-$  linear collider is a very high precision machine in the context of Higgs physics. This precision would allow the determination of the complete profile of the SM Higgs boson, in particular if its mass is smaller than  $\sim 130$   $GeV$ .

Total Production of Higgs Pairs	$e^+e^- \rightarrow b\bar{b}HH \ (t\bar{t}HH)$		
$M_H(GeV)$	$\sqrt{s} = 800 \ GeV$	$\sqrt{s} = 1000 \ GeV$	$\sqrt{s} = 1600 \ GeV$
100	25 (19)	18 (28)	10 (21)
110	26 (13)	19 (22)	10 (18)
120	23 (9)	18 (17)	10 (16)
130	23 (6)	18 (13)	10 (14)
140	21 (3)	18 (19)	10 (12)
150	19 (2)	16 (8)	9 (11)
160	19 (1)	16 (6)	9 (10)
170	17 (-)	15 (4)	9 (9)
180	15 (-)	15 (3)	8 (8)
190	14 (-)	14 (2)	8 (7)
200	12 (-)	13 (1)	8 (6)

Table I. Total production of Higgs pairs in the SM for  $\mathcal{L} = 1000 \ fb^{-1}$   $m_b = 4.5 \ GeV$  and  $m_t = 175 \ GeV$ .

In Fig. 6, we also include a contours plot for the number of events of the studied processes, as a function of  $M_H$  and  $\sqrt{s}$ . These contours are obtained from Table I.

Although the Higgs coupling with top quarks, the largest coupling in the SM, is directly accessible in the process where the Higgs boson is radiated off top quarks  $e^+e^- \rightarrow t\bar{t}HH$ . The coupling with bottom quarks is also accessible in the reaction where the Higgs is radiated by a  $b(\bar{b})$  quark,  $e^+e^- \rightarrow b\bar{b}HH$ . For  $M_H \lesssim 130 \ GeV$ , the Yukawa coupling can be measured with a precision of less than 5% at  $\sqrt{s} = 800 \ GeV$  with a luminosity of  $\mathcal{L} = 1000 \ fb^{-1}$ .

Finally, the measurement of the trilinear Higgs self-coupling, which is the first non-trivial test of the Higgs potential, is accessible in the double Higgs production processes  $e^+e^- \rightarrow b\bar{b}HH$  and in the  $e^+e^- \rightarrow t\bar{t}HH$  process at high energies. Despite its smallness, the cross sections can be determined with an accuracy of the order of 20% at a 800 GeV collider if a high luminosity ( $\mathcal{L} = 1000 \ fb^{-1}$ ) is available.

### III. CONCLUSIONS

In conclusion, the double Higgs production, in association with  $b(\bar{b})$  and  $t(\bar{t})$  quarks ( $e^+e^- \rightarrow b\bar{b}HH, t\bar{t}HH$ ), will be observable at the Next Generation Linear  $e^+e^-$  Colliders. The study of these processes is important in order to know their impact on the three-body process and could be useful to probe anomalous  $HHH$  coupling given the following conditions: very high luminosity, excellent  $b$  tagging performances, center-of-mass large energy, and intermediate range Higgs boson mass.

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## FIGURE CAPTIONS

**Fig. 1** Feynman diagrams at tree-level for  $e^+e^- \rightarrow b\bar{b}HH$ .

**Fig. 2** Total cross section of the Higgs pairs production  $e^+e^- \rightarrow b\bar{b}HH(t\bar{t}HH)$  as function of the Higgs boson mass  $M_H$  for  $\sqrt{s} = 800 \text{ GeV}$  with  $m_b = 4.5 \text{ GeV}$  and  $m_t = 175 \text{ GeV}$ .

**Fig. 3** The same as in Fig. 2, but for  $\sqrt{s} = 1000 \text{ GeV}$ .

**Fig. 4** The same as in Fig. 2, but for  $\sqrt{s} = 1600 \text{ GeV}$ .

**Fig. 5** Total cross section of the Higgs pairs production  $e^+e^- \rightarrow b\bar{b}HH(t\bar{t}HH)$  as a function of the center-of-mass energy  $\sqrt{s}$  for one representative value of the Higgs mass  $M_H = 130 \text{ GeV}$  with  $m_b = 4.5 \text{ GeV}$  and  $m_t = 175 \text{ GeV}$ .

**Fig. 6** Contours plot for the number of events of both processes  $e^+e^- \rightarrow b\bar{b}HH$  and  $e^+e^- \rightarrow t\bar{t}HH$  as a function of  $M_H$  and  $\sqrt{s}$ .

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